Development of Empirical Soil Loss Regression Model (ESLRM)

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This study "Development of Empirical Soil Loss Regression Model" presented an empirical model to estimate soil loss in gullies and steep terrains in a watershed. The properties of soil used for the experiment were determined. Soil loss experiments were conducted in the laboratory and field, using rain simulator on thirty two soil samples. Soil loss was measured and the behavior of the soil during the experiments was studied. The field soil loss experiment was carried out in an artificial gully where runoff was simulated and Check-dams introduced to trap eroded soil. After the experiment, the soil loss was measured. The soil loss parameters, which include rain intensity (I), Slope of the catchment (S), runoff/rain duration (D), density of soil (ρ), Catchment Area (A), Organic content (O), and Clay content (C))were determined. Soil loss model, based on least square approach was developed. The developed model was calibrated using the nine selected observation points. This gave the coefficient of correlation (R) value of 0.99 and R^2 value of 0.99. The R and R^2 values obtained with twenty three observation points (which were not used in the model development) are0.93 and 0.86 . The values when all the thirty two observation points were used are 0.95 and 0.898. The recorded maximum soil loss value is 4.1kg. This model ESLRM will serve as useful tools in planning and design of erosion and sediment control projects in a basin and as powerful tool for soil loss estimation. Finally, it is recommended that further studies be carried out on the universal applicability of the model.

Key words: calibration, catchment, empirical, model, parameters, soil loss, verification.

1. INTRODUCTION

The need for empirical soil loss models for different watersheds in the world to predict the rate of erosion can never be overemphasized. Taking steps to preserve the quality and quantity of global soil resources cannot also be overemphasized. Our future ability to feed ourselves and to live in an unpolluted environment depends on our ability to understand and know how to reduce the rates at which our soils are currently eroding. The model will predict the soil loss and also show the parameters that have contributed to their success. It is hoped that the results presented herein would be of interest to planners and designers of gully control measures in Nigeria and else wherein which similar gully erosion problems occur.This gave credence to the fact that soil is gradually being lost from the surface of the earth. Thus, to mitigate against these menace of road surface water runoff and gully erosion, it becomes pertinent to know the geological and hydrological behaviour of soils inany place. This knowledge, when put in form of model will go a long way in helping government policy makers to check the problems of erosion.This study concentrated on development of Empirical Soil Loss Regression Model (ESLRM).

The parameters used in the model development includes, rainfall intensity, slope of the catchment, catchment area, duration of experiment, density of soil, organic matter content in the soil, and clay content in the soil. The empirical soil loss regression model (ESLRM) will be most suitable for soil parameters/characteristics irrespective of the slope, rainfall intensity, rainfall duration and catchment area size.

2.1 Materials for Soil Loss Simulation

KAMPHUST Rainfall simulator (Fig. 1&2), water, sandy soil, measuring cylinders, stop watch, weighing scale, plastic bowl, oven, scoop, spade, soil bin, rice husk.

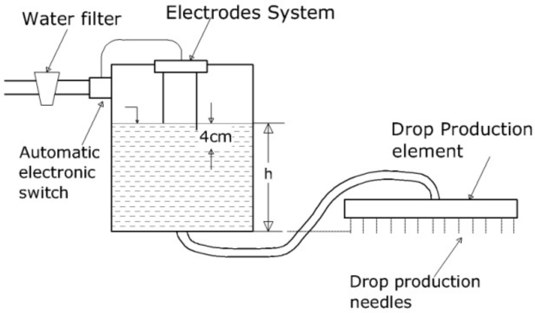

Fig. 1: Diagrammatic representation of rain simulators drop production system
(Mutchler and Hermsmeier, 1995)

The rainfall simulator is a device used to apply water in form or rainfall on the soil sample. It is made up of a plastic bucket with a tap on the base and a flexible delivery tube attached to the bucket tap to convey the water to the perforated glass through which it rains. The perforated glass is held with a four legged frame on which the bucket of water is placed.

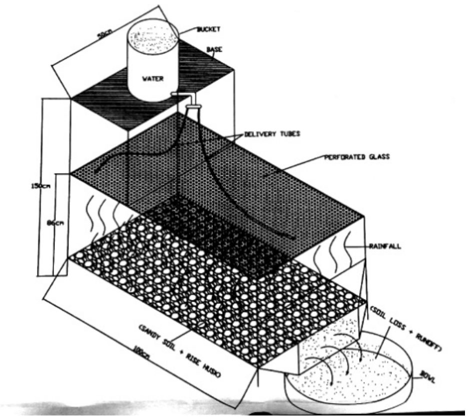

Fig. 2: Diagram of a Rainfall Simulator

The bucket is manually filled to the brim from time to time to ensure a steady pressure. The water used was gotten from the public water source; sandy soil used was a bit moist before mixing it with the organic matter.

The cylinders were used for measuring the volume of runoff while the stop watch was used to time the runoff. The scale was used to measure the weights of both the rice husk and the sandy soil before they were mixed together thoroughly and the plastic bowl was used to collect the runoff while the oven was used to dry the soil loss so as to determine its moist and dry weights.

The rice husk is the dry outer covering of rice and was gotten from the rice mill. It was kept for a period of one week before usage to make sure that there was a proper breakdown.

2.1.1 Determination of Soil properties

Here different types of soil sample used for the experiment were examine and their properties were determine, the soil properties includes; clay content, density and organic content were determined. The density test of soil was done in accordance to ASTM 2974 method, (Garg, 2013).

2.1.2 Organic Content Determination

This was done in accordance to ASTM 2974 method of organic matter content of soils. The soil sample was firstly oven dried at 100^0c. It was weighed before putting in a porcelain dish. At this point, it was weighed together with porcelain dish and placed in a furnace at a temperature of 440^0c for 2 hours. It was brought out of the furnace and allowed to cool to room temperature before weighing it together with the porcelain again, (Garg, 2013).

2.1.3 Clay Content Determination

The Soil Sample of about 150g is collected from site and taken to the laboratory. The soil is dissolved in a beaker about (100ml) and stirred vigorously and allowed to settle (24hrs) and the soil layered,(Venkatramaiah, 2012).

2.1.4 Simulation of Soil Loss in a Catchment

Soil loss experiments were conducted in the laboratory using a rain simulator, on thirty-two times with three different tray catchment and gully channels these soil sample from different sites. Soil loss was measured and the attitude of soils during the experiments was studied. Samples of disturbed soil were used in the experiments.

2.1.5 Rain Simulator

The constructed rain simulator belongs in the category of those where the rain drops form and fall from protruding needles, starting from zero velocity.

The needles used here had an internal diameter of 0.58 mm and were set at the corners of squares with side dimension of 2.5 cm (Chow and Harbaugh, 1995). Drop production is a result of hydrostatic pressure, h, that is gained by connecting the drop production system to a water container where the water level is kept at a constant height (± 2 cm) with an automated system of electrodes. In this way, the produced rain has a constant intensity.

Different rain intensity and drop size can be achieved by changing the hydrostatic pressure by moving the electrode system vertically. The produced raindrops fall on soil bed after travelling a distance of 7.5 m, when they have obtained at least 95% of their maximum kinetic energy or their terminal velocity (Morgan et al., 1998). The soil sample is placed on a rectangular shaped bed sizing 1.90 x 0.60 x 0.20 m, 1 x 0.5 x 0.20 m, and 7.5 x 0.3 x 0.5 m. Using a pulley, the soil bed slope can be changed. For soil loss experiments, rain intensity of 28 mm h^{-1} with a drop diameter of 2.58 mm and soil slope of 5% were used.

The duration of every experiment was 4 hours.

The following values were measured in the cause of each of the experiment.

i. Duration (time),
ii. Area of catchment or rectangular shaped bed (meters)
iii. Volume of water used (runoff)
iv. Volume of soil loss
v. Mass of soil loss (we be one of the calculated variables)

$$MassofSoilLoss = V * \rho \qquad (1)$$

Where, V = volume of soil loss, ρ = bulk density of soil

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$$Rainfall Intensity I = \frac{(V_{ml} * 1000) * mm^2}{Area(mm^2)} / Time(min) \quad (2)$$

Where V_{ml} = volume of water used (runoff) in milliliter, Area = Area of catchment or the rectangular shape plate in millimeter square, Time = time or the duration period of the rainfall or water over the prepared soil in minuet.

$$Velocity V = 0.125\sqrt{2gH} \quad (3)$$

Where, H is height of water tank, g is acceleration due to gravity.

$$Discharge Q = AV \quad (4)$$

Where A is area of nozzle and V is velocity,

$$Volume V_{ml} = Q * T = \frac{m^3}{s} * s = m^3 \quad (5)$$

Where, $V_{ml} = m^3 * 1000 * 1000 = V_{ml}$, and T is time or duration of rainfall.

2.1.6 Run-off Simulator through gully channel

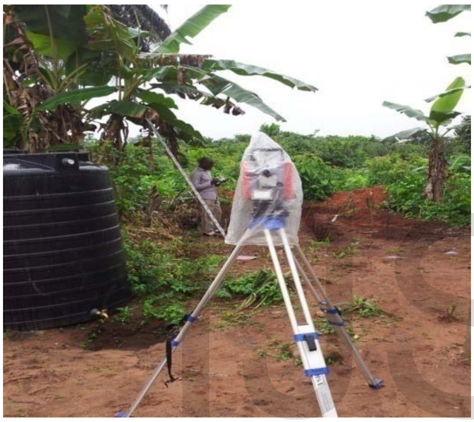

Plate 1: The theodolite on the tripod stand, set for the experiment

Plate 2: The excavation of artificial gully and inserted metal plates (check dam) ready for experiment.

For slope, to find the average change in elevation over the total length of the channel and mathematical expressed in Eqn.(6),

$$S = \frac{\Delta elevation}{Length} * 100 \quad (6)$$

A horizontal run-off though gully channel experiment was carried out by construction of an Artificial gully channel of 7.5 x 0.3 x 0.5 m with different slope of 5%, 8, and 9%. Were run-off simulator was used through the gully channel with check dams of 1.5m interval in the channel was used to trap soil loss between the check dams. Each of the experiment takes four hours and by consequence two experiments was carried out in a day, for any duration of the experiment. These types of experiment moves soil particle which can be movable by the strength of the water current and has the ability of nature soil resistance base on the soil type. The slope is as a result of different cause in each of the experiment and Plate 1, 2 and 3shows the typical constructed gully, the check dam and the theodolite that is surveyor instrument used in the leveling by which the slope is computed each time before any experiment will start.

Plate 3: Experiment in progress, the water is being pumped from the tank, pressurized with the help of pressure pump of 0.5 horse power (hp)

2.2 Formulation of Empirical Soil Loss Regression Model

The following parameters have been observed to have effect on soil loss and known to be the operation function of soil loss: rainfall intensity I, Slope of the catchment S, Duration of rainfall D, bulk density ρ, Area of catchment A, Organic Mater O, and Clay content C.

Applying the multiply regression the variables can relate with the soil loss as;

$$SoilLoss = S_L = e^{\alpha_0} * I^{\alpha_1} * S^{\alpha_2} * D^{\alpha_3} * \rho^{\alpha_4} * A^{\alpha_5} * O^{\alpha_6} * C^{\alpha_7} \quad (7)$$

Taking the natural logarithm of both the left and right hand side of Eqn.(7)made it a linear function as:

$$\ln S_L = \alpha_0 + \alpha_1 \ln I + \alpha_2 \ln S + \alpha_3 \ln D + \alpha_4 \ln \rho + \alpha_5 \ln A + \alpha_6 \ln O + \alpha_7 \ln C \quad (8)$$

Replacing $\ln S_L, \ln I, \ln S, \ln D, \ln \rho, \ln A, \ln O$ and $\ln g$ gives:
by y x_1, x_2, x_3, x_4, x_5, x_6 and x_7 respectively

$$y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5 + \alpha_6 x_6 + \alpha_7 x_7. That is:$$

$$y = \alpha_0 + \sum_{i=1}^{n=7} \alpha_i x_i \qquad (9)$$

In order to obtain α_i (i = 0 to 7) through least square method the Eqns. (10) – (17) were obtain (Agunwamba, 2007):

$$\sum y = n\alpha_0 + \alpha_1 \sum x_1 + \alpha_2 \sum x_2 + \alpha_3 \sum x_3 + \alpha_4 \sum x_4 + \alpha_5 \sum x_5 + \alpha_6 \sum x_6 + \alpha_7 \sum x_7 \qquad (10)$$

$$\sum x_1 y = \alpha_0 \sum x_1 + \alpha_1 \sum x_1^2 + \alpha_2 \sum x_1 x_2 + \alpha_3 \sum x_1 x_3 + \alpha_4 \sum x_1 x_4 + \alpha_5 \sum x_1 x_5 + \alpha_6 \sum x_1 x_6 + \alpha_7 \sum x_1 x_7 \qquad (11)$$

$$\sum x_2 y = \alpha_0 \sum x_2 + \alpha_1 \sum x_1 x_2 + \alpha_2 \sum x_2^2 + \alpha_3 \sum x_2 x_3 + \alpha_4 \sum x_2 x_4 + \alpha_5 \sum x_2 x_5 + \alpha_6 \sum x_2 x_6 + \alpha_7 \sum x_2 x_7 \qquad (12)$$

$$\sum x_3 y = \alpha_0 \sum x_3 + \alpha_1 \sum x_1 x_3 + \alpha_2 \sum x_2 x_3 + \alpha_3 \sum x_3^2 + \alpha_4 \sum x_3 x_4 + \alpha_5 \sum x_3 x_5 + \alpha_6 \sum x_3 x_6 + \alpha_7 \sum x_3 x_7 \qquad (13)$$

$$\sum x_4 y = \alpha_0 \sum x_4 + \alpha_1 \sum x_1 x_4 + \alpha_2 \sum x_2 x_4 + \alpha_3 \sum x_3 x_4 + \alpha_4 \sum x_4^2 + \alpha_5 \sum x_4 x_5 + \alpha_6 \sum x_4 x_6 + \alpha_7 \sum x_4 x_7 \qquad (14)$$

$$\sum x_5 y = \alpha_0 \sum x_5 + \alpha_1 \sum x_1 x_5 + \alpha_2 \sum x_2 x_5 + \alpha_3 \sum x_3 x_5 + \alpha_4 \sum x_4 x_5 + \alpha_5 \sum x_5^2 + \alpha_6 \sum x_5 x_6 + \alpha_7 \sum x_5 x_7 \qquad (15)$$

$$\sum x_6 y = \alpha_0 \sum x_6 + \alpha_1 \sum x_1 x_6 + \alpha_2 \sum x_2 x_6 + \alpha_3 \sum x_3 x_6 + \alpha_4 \sum x_4 x_6 + \alpha_5 \sum x_5 x_6 + \alpha_6 \sum x_6^2 + \alpha_7 \sum x_6 x_7 \qquad (16)$$

$$\sum x_7 y = \alpha_0 \sum x_7 + \alpha_1 \sum x_1 x_7 + \alpha_2 \sum x_2 x_7 + \alpha_3 \sum x_3 x_7 + \alpha_4 \sum x_4 x_7 + \alpha_5 \sum x_5 x_7 + \alpha_6 \sum x_6 x_7 + \alpha_7 \sum x_7^2 \qquad (17)$$

Eqns.(10)to(17)can be summarized in matrix equations as shown on Eqn.(18).

$$\begin{bmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \\ \sum x_3 y \\ \sum x_4 y \\ \sum x_5 y \\ \sum x_6 y \\ \sum x_7 y \end{bmatrix} = \begin{Bmatrix} n & \sum x_1 & \sum x_2 & \sum x_3 & \sum x_4 & \sum x_5 & \sum x_6 & \sum x_7 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 & \sum x_1 x_3 & \sum x_1 x_4 & \sum x_1 x_5 & \sum x_1 x_6 & \sum x_1 x_7 \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 & \sum x_2 x_3 & \sum x_2 x_4 & \sum x_2 x_5 & \sum x_2 x_6 & \sum x_2 x_7 \\ \sum x_3 & \sum x_1 x_3 & \sum x_2 x_3 & \sum x_3^2 & \sum x_3 x_4 & \sum x_3 x_5 & \sum x_3 x_6 & \sum x_3 x_7 \\ \sum x_4 & \sum x_1 x_4 & \sum x_2 x_4 & \sum x_3 x_4 & \sum x_4^2 & \sum x_4 x_5 & \sum x_4 x_6 & \sum x_4 x_7 \\ \sum x_5 & \sum x_1 x_5 & \sum x_2 x_5 & \sum x_3 x_5 & \sum x_4 x_5 & \sum x_5^2 & \sum x_5 x_6 & \sum x_5 x_7 \\ \sum x_6 & \sum x_1 x_6 & \sum x_2 x_6 & \sum x_3 x_6 & \sum x_4 x_6 & \sum x_5 x_6 & \sum x_6^2 & \sum x_6 x_7 \\ \sum x_7 & \sum x_1 x_7 & \sum x_2 x_7 & \sum x_3 x_7 & \sum x_4 x_7 & \sum x_5 x_7 & \sum x_6 x_7 & \sum x_7^2 \end{Bmatrix} * \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{bmatrix} \qquad (18)$$

Rearranging Eqn.(18) and making the unknown coefficient subject of the equation gives:

$$\begin{bmatrix}\alpha_0\\\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\\\alpha_5\\\alpha_6\\\alpha_7\end{bmatrix}=\left\{\begin{matrix} n & \sum x_1 & \sum x_2 & \sum x_3 & \sum x_4 & \sum x_5 & \sum x_6 & \sum x_7\\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 & \sum x_1x_3 & \sum x_1x_4 & \sum x_1x_5 & \sum x_1x_6 & \sum x_1x_7\\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 & \sum x_2x_3 & \sum x_2x_4 & \sum x_2x_5 & \sum x_2x_6 & \sum x_2x_7\\ \sum x_3 & \sum x_1x_3 & \sum x_2x_3 & \sum x_3^2 & \sum x_3x_4 & \sum x_3x_5 & \sum x_3x_6 & \sum x_3x_7\\ \sum x_4 & \sum x_1x_4 & \sum x_2x_4 & \sum x_3x_4 & \sum x_4^2 & \sum x_4x_5 & \sum x_4x_6 & \sum x_4x_7\\ \sum x_5 & \sum x_1x_5 & \sum x_2x_5 & \sum x_3x_5 & \sum x_4x_5 & \sum x_5^2 & \sum x_5x_6 & \sum x_5x_7\\ \sum x_6 & \sum x_1x_6 & \sum x_2x_6 & \sum x_3x_6 & \sum x_4x_6 & \sum x_5x_6 & \sum x_6^2 & \sum x_6x_7\\ \sum x_7 & \sum x_1x_7 & \sum x_2x_7 & \sum x_3x_7 & \sum x_4x_7 & \sum x_5x_7 & \sum x_6x_7 & \sum x_7^2\end{matrix}\right\}^{-1} * \begin{bmatrix}\sum y\\\sum x_1y\\\sum x_2y\\\sum x_3y\\\sum x_4y\\\sum x_5y\\\sum x_6y\\\sum x_7y\end{bmatrix} \quad (19)$$

After the coefficients were determined, they would be substituted back into the original Eqn.(7) for soil loss prediction of any catchment with known parameters over any rainfall intensity and duration of the rainfall. The Eqn.(7) is reproduced here as:

$$Soil\ Loss = SL = e^{\alpha_0} * I^{\alpha_1} * S^{\alpha_2} * D^{\alpha_3} * \rho^{\alpha_4} * A^{\alpha_5} * O^{\alpha_6} * C^{\alpha_7} \quad (20)$$

Where e is exponential constant and is taken as 2.718281828

The determination of the coefficients was done below, after the Empirical laboratory results were obtained.

3.1 Total Soil Loss from Experimental Catchment

The soil loss measured empirically in the laboratory and the corresponding soil characteristics, catchment size, catchment slope, Rainfall intensity and rainfall duration are presented in Table 1.

Table 1: Experimental Soil Loss Values and its Parameters

S_L (kg)	I (mm/min)	S_o (%)	D (s)	ρ (kN/m³)	A (m²)	O (%)	C (%)	E.No
0.612	4.99	7.5	305	19.8	2.25	1.2	10	1
0.576	4.9	10.03	300	19.9	2.25	2.1	13	2
0.542	3.92	11.98	240	20.1	2.25	1.8	11	3
2.86	4.9	10	1800	20.1	0.5	5.2	10	4
1.508	5.1	10	1800	19.9	0.5	9.2	8.1	5
1.2	5.3	10	1800	18.9	0.5	13.2	6.9	6
3.9	5.4	10	1800	18.9	0.5	1.2	12	7
0.575	5.3	10	1800	19.8	0.5	17.2	5.8	8
3.75	5	7.5	1830	19.9	2.25	1.2	9	9
3.56	5.1	10.1	1800	19.8	2.25	2	10	10
3.34	4.1	11.1	1440	19.9	2.25	1.2	9	11
0.54	5.1	10	360	18.73	0.5	5.3	10	12
0.31	5	10	370	18.3	0.5	9.2	8	13
0.24	5.2	10	365	17.5	0.5	13	11	14
0.81	5.3	10	364	19.9	0.5	1.2	12	15
0.12	5.2	10	370	17.4	0.5	18	13	16
1.88	5.1	7.5	915	19.9	2.25	1.2	9	17
1.77	5.1	10.1	900	19.8	2.25	2.1	11	18
1.66	4.1	11.1	720	19.9	2.25	1.2	10	19
0.323	7.02	5	6900	18.55	1.14	30.22	45.8	20
0.518	12.28	5	5400	19	1.14	50.04	61.7	21
0.66	14.04	5	5100	18.5	1.14	60.42	57.7	22
0.304	7.02	5	6000	19.8	1.14	16.59	46.1	23
0.162	7.02	5	3450	18.5	1.14	30.1	45.7	24
0.259	12.28	5	2700	19.5	1.14	50.04	61.7	25
0.331	14.04	5	2550	18.7	1.14	60.42	57.7	26
0.152	7.02	5	3000	19.8	1.14	16.59	46.1	27
1.43	4.95	10	900	19.9	0.5	6	11	28
0.754	5.1	10	900	19.9	0.5	9.2	10	29
0.88	4.1	11.1	360	19.9	2.25	1.3	10	30
0.94	5.1	7.5	458	19.9	2.25	1.2	10	31
1.95	5.4	10	900	18.9	0.5	1.3	11	32

Legend:
S_L = soil loss; I = rainfall intensity; S_o = slope of the catchment;
D = duration of the experiment; ρ = density of soil; A = catchment area; O = organic matter content in the soil; C = clay content in the soil; E.No = Experimental Number.

3.2 Empirical Soil Loss Regression Model and Experimental

The results of experimental soil loss and their parameters were showed in Table 1, which is the soil loss from Rainfall and Run-off Simulator. The empirical soil loss model result was further compared with the setup laboratory soil loss test result. To obtain the Soil loss regression model of any catchment, nine experimental result was selected with deeply and peculiar cases.

i. The experiment with highest and lowest soil loss was selected.
ii. The experiment with highest and lowest duration for different catchment try was selected and.
iii. The experiment with lowest and highest organic and clay content was also selected.

The nine (9) selected experimental used in soil loss model regression was highlighted in Table 1. Eqn. 20 is the soil loss exponential equation of this study, putting the soil loss S_L, and other parameters of the experiment as its functions as showed in Eqn. (7) of polynomial expression. Which was linearized as showed in Eqn. (8) using natural logarithm and later employed least square method by (Agunwamba, 2007) in solving the equation as follows.
The data selected from Table 1, were used for soil loss model regression by employing the method discussed under section 2.2. The measured parameters were re-tabulated in Table 2 and solved with the help of matrix in Eqn. (21 & 22) in order to determine the coefficient in the equation and substituted in Eqn.(7).

Table 2: Tabulation of Matrix Parameters

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_1y	x_1^2	x_1x_2	x_1x_3	x_1x_4	x_1x_5	x_1x_6	x_1x_7	x_2y	x_2^2	x_2x_3	x_2x_4	x_2x_5	x_2x_6
	In I	In So	In D	In ρ	In A	In O	In C														
	1.607	2.015	5.720	2.986	0.811	0.182	2.303	-0.788	2.584	3.239	9.195	4.799	1.304	0.293	3.701	-0.988	4.060	11.526	6.016	1.634	0.367
	1.668	2.303	7.496	2.939	-0.693	2.580	1.932	0.304	2.781	3.840	12.500	4.902	-1.156	4.303	3.221	0.420	5.302	17.259	6.768	-1.596	5.941
	1.686	2.303	7.496	2.939	-0.693	0.182	2.485	2.295	2.844	3.883	12.640	4.957	-1.169	0.307	4.191	3.134	5.302	17.259	6.768	-1.596	0.420
	2.642	1.609	8.537	2.918	0.131	4.101	4.055	-1.098	6.980	4.252	22.554	7.708	0.346	10.835	10.714	-0.669	2.590	13.740	4.696	0.211	6.601
	1.949	1.609	8.700	2.986	0.131	2.809	3.831	-2.320	3.798	3.136	16.953	5.818	0.255	5.474	7.465	-1.916	2.590	14.001	4.805	0.211	4.521
	2.508	1.609	7.901	2.970	0.131	3.913	4.122	-3.388	6.290	4.036	19.816	7.450	0.329	9.813	10.339	-2.174	2.590	12.716	4.781	0.211	6.297
	1.649	2.303	5.914	2.856	-0.693	2.890	2.565	-3.496	2.718	3.796	9.749	4.709	-1.143	4.765	4.229	-4.882	5.302	13.616	6.577	-1.596	6.655
	1.411	2.407	5.886	2.991	0.811	0.262	2.303	-0.180	1.991	3.396	8.305	4.220	1.144	0.370	3.249	-0.308	5.793	14.168	7.198	1.952	0.631
	1.609	2.303	5.914	2.907	-0.693	2.219	2.079	-1.885	2.590	3.706	9.517	4.678	-1.116	3.572	3.347	-2.697	5.302	13.616	6.693	-1.596	5.110
Σ	16.729	18.461	63.562	26.492	-0.758	19.140	25.674	-10.556	32.576	33.285	121.231	49.242	-1.205	39.733	50.455	-10.080	38.832	127.902	54.302	-2.166	36.544

Table 2 contd.

x_2x_7	x_3y	x_3^2	x_3x_4	x_3x_5	x_3x_6	x_3x_7	x_4y	x_4^2	x_4x_5	x_4x_6	x_4x_7	x_5y	x_5^2	x_5x_6	x_5x_7	x_6y	x_6^2	x_6x_7	x_7y	x_7^2
4.639	-2.805	32.722	17.079	4.639	1.043	13.172	-1.464	8.914	2.421	0.544	6.875	-0.398	0.658	0.148	1.867	-0.089	0.033	0.420	-1.129	5.302
4.447	1.367	56.183	22.031	-5.196	19.340	14.478	0.536	8.639	-2.037	7.584	5.677	-0.126	0.480	-1.788	-1.339	0.470	6.658	4.984	0.352	3.731
5.722	10.201	56.183	22.031	-5.196	1.367	18.626	4.000	8.639	-2.037	0.536	7.304	-0.943	0.480	-0.126	-1.722	0.248	0.033	0.453	3.382	6.175
6.527	-3.547	72.880	24.909	1.119	35.013	34.620	-1.212	8.513	0.382	11.967	11.832	-0.054	0.017	0.537	0.531	-1.704	16.821	16.632	-1.685	16.445
6.165	-10.359	75.682	25.974	1.140	24.435	33.326	-3.555	8.914	0.391	8.386	11.438	-0.156	0.017	0.368	0.502	-3.345	7.889	10.760	-4.561	14.675
6.635	-10.674	62.426	23.469	1.035	30.915	32.570	-4.013	8.823	0.389	11.623	12.245	-0.177	0.017	0.513	0.540	-5.286	15.310	16.130	-5.569	16.993
5.906	-12.538	34.970	16.892	-4.099	17.092	15.168	-6.056	8.159	-1.980	8.256	7.327	1.470	0.480	-2.003	-1.778	-6.128	8.354	7.414	-5.438	6.579
5.542	-0.752	34.646	17.604	4.773	1.544	13.553	-0.382	8.944	2.425	0.785	6.886	-0.104	0.658	0.213	1.867	-0.034	0.069	0.604	-0.294	5.302
4.788	-6.926	34.970	17.190	-4.099	13.123	12.297	-3.405	8.450	-2.015	6.451	6.045	0.812	0.480	-1.538	-1.441	-2.599	4.925	4.615	-2.435	4.324
50.372	-36.033	460.661	187.178	-5.883	143.873	187.809	-15.552	77.997	-2.060	56.131	75.628	0.323	3.289	-3.678	-0.973	-18.466	60.092	62.011	-17.379	79.526

The summation of the tabulated data and the total number of the experiment nine (9) n is used in the method matrix as follows;

$$\begin{bmatrix} -5.32352 \\ -10.5562 \\ -10.0804 \\ -36.0333 \\ -15.5517 \\ 0.322932 \\ -18.4664 \\ -17.3786 \end{bmatrix} = \begin{pmatrix} 9 & 16.72927073 & 18.4605 & 63.56202 & 26.49196 & -0.75764 & 19.13974 & 25.67434 \\ 16.72927073 & 32.57557937 & 33.28498 & 121.2306 & 49.24183 & -1.20538 & 39.73294 & 50.45493 \\ 18.4605 & 33.28498 & 38.83168 & 127.9016 & 54.30229 & -2.16567 & 36.54392 & 50.37169 \\ 63.56202 & 121.2306 & 127.9016 & 460.6613 & 187.1779 & -5.88316 & 143.8728 & 187.809 \\ 26.49196 & 49.24183 & 54.30229 & 187.1779 & 77.99659 & -2.06024 & 56.13143 & 75.62799 \\ -0.75764 & -1.20538 & -2.16567 & -5.88316 & -2.06024 & 3.288533 & -3.67781 & -0.97258 \\ 19.13974 & 39.73294 & 36.54392 & 143.8728 & 56.13143 & -3.67781 & 60.09232 & 62.01075 \\ 25.67434 & 50.45493 & 50.37169 & 187.809 & 75.62799 & -0.97258 & 62.01075 & 79.52584 \end{pmatrix} * \begin{bmatrix} b \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{bmatrix} \quad - 21$$

To solve for the unknown parameters by making them subject of the mater. We have ;

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{bmatrix} = \begin{pmatrix} 3323.328 & -41.30797 & -87.4303 & 23.8887 & -1063.63 & 50.43911 & -9.40275 & -28.2943 \\ -41.30797 & 5.4349334 & 1.779599 & -0.42289 & 11.45578 & -0.55641 & -0.32246 & -0.89035 \\ -87.4303 & 1.779599 & 11.41991 & 0.18094 & 17.55762 & 0.592482 & 0.480873 & 2.371705 \\ 23.8887 & -0.42289 & 0.18094 & 0.524964 & -8.86438 & 0.751091 & 0.004262 & -0.36259 \\ -1063.63 & 11.45578 & 17.55762 & -8.86438 & 353.1717 & -18.404 & 2.89714 & 7.585301 \\ 50.43911 & -0.55641 & 0.592482 & 0.751091 & -18.404 & 1.860204 & 0.112317 & -0.64279 \\ -9.40275 & -0.32246 & 0.480873 & 0.004262 & 2.89714 & 0.112317 & 0.232798 & -0.00975 \\ -28.2943 & -0.89035 & 2.371705 & -0.36259 & 7.585301 & -0.64279 & -0.00975 & 1.852332 \end{pmatrix} * \begin{bmatrix} -5.32352 \\ -10.5562 \\ -10.0804 \\ -36.0333 \\ -15.5517 \\ 0.322932 \\ -18.4664 \\ -17.3786 \end{bmatrix} = \begin{bmatrix} -12.2867 \\ 2.921707 \\ 2.057649 \\ 0.87341 \\ 0.074376 \\ 0.234785 \\ -0.68998 \\ -1.00134 \end{bmatrix} \quad - 22$$

From the above solution b is used to replaced Inα_0 so α_0 will determined while α_1 to $\alpha_7$was already known and they will be substitute back into Eqn.(20) which will yield soil loss prediction equation for any catchment. From this equation, b is-12.2867 and the equation coefficientsα0 to α7 are -12.2867, 2.921707, 2.057649, 0.87341, 0.074376, 0.234785, -0.68998 and -1.00134 respectively. And the Soil Loss model regression as shown in Eqn. (23) haven substituted the coefficient of α_0 to α_7.

3.3 Empirical Soil Loss Regression Model (ESLRM)

The empirical soil loss regression model (ESLRM) formulated in this study is presented herein as;

$$S_L = e^{(-12.2867)} * I^{(2.921707)} * S^{(2.057649)} * D^{(0.87341)} * \rho^{(0.074376)} * A^{(0.234785)} * O^{(-0.68998)} * C^{(-1.00134)} \qquad (23)$$

4.1Calibration of ESLRM and Experimental Soil Loss Measured

Soil losses predicted from ESLRM and Experimental Measured Soil Loss were plotted as presented on Fig.1.

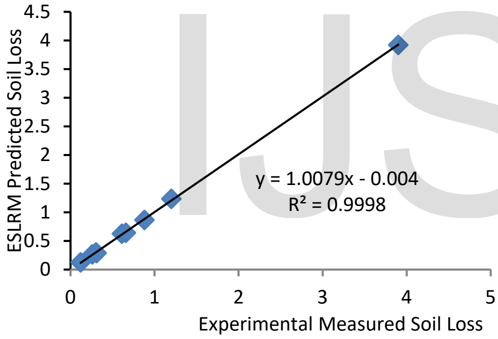

Fig. 1: Calibration of Measured and Predicted Soil Loss Using Empirical Model (ESLRM)

Looking at Fig. 1, you see that calibration of measured and predicted soil loss using ESLRM give a perfect bisect line of Angle 45^0 linear. The soil loss graphical yield mode or equation from Fig. 1, as $Y = 1.007x - 0.004$ with a mean square root$R^2 = 0.999$ and the figure gives a positive slope graph.

4.2: Correlation (r) of Empirical Soil Loss Regression Model

This Eqn. (23) is used for soil loss prediction with the experimental catchment and the result of the prediction and the measured in Table 3 were compared with correlation which is shown below as follows;

Table 3: Selected Experimental Soil loss Measured and Predicted

Experimental number	Measured values	Predicted values
1	0.612	0.626810386
6	1.2	1.236026246
7	3.9	3.923121098
22	0.66	0.642365526
23	0.304	0.299927005
25	0.259	0.266455679
16	0.12	0.124936709
30	0.88	0.865505989
13	0.31	0.288939631

Taken the measured values of the soil loss as independent variable x and predicted values as the dependent variable y, we can use the coefficient of correlation (r) relationship tabulated in Table 4 as follows;

Table 4: Tabulation for correlation coefficient (r) relation

	X	Y	XY	X^2	Y^2
	0.612	0.62681	0.3836	0.3745	0.3929
	1.2	1.23603	1.4832	1.4400	1.5278
	3.9	3.92312	15.3002	15.2100	15.3909
	0.66	0.64237	0.4240	0.4356	0.4126
	0.304	0.29993	0.0912	0.0924	0.0900
	0.259	0.26646	0.0690	0.0671	0.0710
	0.12	0.12494	0.0150	0.0144	0.0156
	0.88	0.86551	0.7616	0.7744	0.7491
	0.31	0.28894	0.0896	0.0961	0.0835
Σ	8.245	8.274	18.617	18.505	18.733

Invoking Equation for correlation

$$r_{xy} = \frac{n\sum xy - (\sum y)(\sum x)}{\sqrt{[n\sum y^2 - (\sum y)^2][n\sum x^2 - (\sum x)^2]}} \qquad 24$$

We substitute the function of the Eqn. (24) will yield;

$$r_{xy} = \frac{A}{\sqrt{B.C}}$$

Where:

$A = 9 * 18.617 - (8.274)(8.245)$

$B = [9 * 18.733 - (8.274)^2]$

$C = [9 * 18.505 - (8.245)^2]$

$$r_{9,selected\ result} = 0.9998544$$

The nine selected experimental results that were used for the regression model for the soil loss prediction were checked using the coefficient of correlation, r to confirm the goodness of fit of the two results of soil loss values. That is the measured and predicted ones, and their coefficient of Correlation, r = 0.9998544.

4.3 Verification of Empirical Soil Loss Regression (ESLRM) Model

The results shown in Table 5 which is the entire experimental result of soil loss measured and predicted with ESLRM of Eqn. (23) were plotted as shown in Fig. 2.

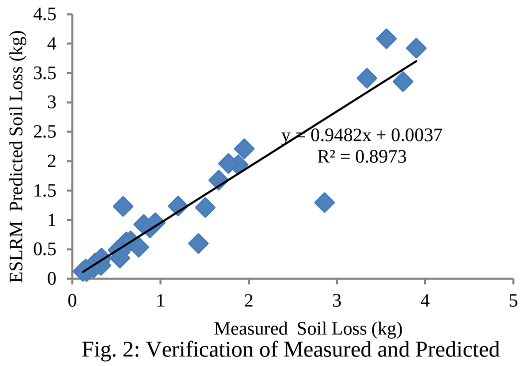

Fig. 2: Verification of Measured and Predicted Soil Loss Using Empirical Model for all the 32 test

Fig. 2, you see that verification of measured and predicted soil loss using ESLRM for the entire experiment, give a perfect bisect line of Angle 45^0 linear. The soil loss graphical yield equation is as $Y = 0.948x + 0.003$ with a mean square root$R^2 = 0.897$ and the figure gives a positive slope graph.

The results of the measured and predicted soil loss values using ESLRM model as presented in Table 5 for the non-highlighted 23 other experimental results were used for the verification of the ESLRM model as presented in Fig.3.

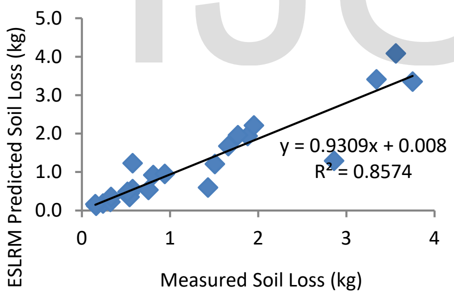

Figure 3: Verification of Measured and Predicted Soil Loss Using Empirical Model for other 23 test

While Fig. 3, responded inline with the Fig. 2 of the entire experiment. And it soil loss graphical yield equation is as $Y = 0.930x + 0.008$ with a mean square root$R^2 = 0.86$ and the figure gives a positive slope graph.

Table 5: Predicted and Measured Soil Loss of all Experimental

Experimental number	Measured values	Predicted values
1	0.612	0.626810386
2	0.576	0.557062637
3	0.542	0.452941138
4	2.86	1.294862586
5	1.508	1.211529407
6	1.2	1.236026246
7	3.9	3.923121098
8	0.575	1.229521662
9	3.75	3.351992997
10	3.56	4.083942717
11	3.34	3.411294847
12	0.54	0.35035651
13	0.31	0.288939631
14	0.24	0.182758038
15	0.81	0.923172781
16	0.12	0.124936709
17	1.88	1.938686902
18	1.77	1.95924474
19	1.66	1.675628907
20	0.323	0.224418437
21	0.518	0.487201428
22	0.66	0.642365526
23	0.304	0.299927005
24	0.162	0.123081152
25	0.259	0.266455679
26	0.331	0.350919049
27	0.152	0.163716692
28	1.43	0.599144049
29	0.754	0.53551759
30	0.88	0.865505989
31	0.94	0.953192442
32	1.95	2.210869686

Correlation, r = 0.947279

However, Table 6 shows the tabulation of parameters of correlation coefficient. From Table 5 measured soil loss values were represented with x and predicted soil loss values were represented as y, as obtain in Table 6.

Table 6: Parameters for correlation

	x	y	xy	X^2	Y^2
	0.612	0.6268	0.3836	0.3745	0.3929
	0.576	0.5571	0.3209	0.3318	0.3103
	0.542	0.4529	0.2455	0.2938	0.2052
	2.86	1.2949	3.7033	8.1796	1.6767
	1.508	1.2115	1.8270	2.2741	1.4678
	1.2	1.2360	1.4832	1.4400	1.5278
	3.9	3.9231	15.3002	15.2100	15.3909
	0.575	1.2295	0.7070	0.3306	1.5117
	3.75	3.3520	12.5700	14.0625	11.2359
	3.56	4.0839	14.5388	12.6736	16.6786
	3.34	3.4113	11.3937	11.1556	11.6369
	0.54	0.3504	0.1892	0.2916	0.1227
	0.31	0.2889	0.0896	0.0961	0.0835
	0.24	0.1828	0.0439	0.0576	0.0334
	0.81	0.9232	0.7478	0.6561	0.8522
	0.12	0.1249	0.0150	0.0144	0.0156
	1.88	1.9387	3.6447	3.5344	3.7585
	1.77	1.9592	3.4679	3.1329	3.8386
	1.66	1.6756	2.7815	2.7556	2.8077
	0.323	0.2244	0.0725	0.1043	0.0504
	0.518	0.4872	0.2524	0.2683	0.2374
	0.66	0.6424	0.4240	0.4356	0.4126
	0.304	0.2999	0.0912	0.0924	0.0900
	0.162	0.1231	0.0199	0.0262	0.0151
	0.259	0.2665	0.0690	0.0671	0.0710
	0.331	0.3509	0.1162	0.1096	0.1231
	0.152	0.1637	0.0249	0.0231	0.0268
	1.43	0.5991	0.8568	2.0449	0.3590
	0.754	0.5355	0.4038	0.5685	0.2868
	0.88	0.8655	0.7616	0.7744	0.7491
	0.94	0.9532	0.8960	0.8836	0.9086
	1.95	2.2109	4.3112	3.8025	4.8879
∑	38.42	36.5451	81.7521	86.0653	81.7647

For coefficient of correlation substitute the function of the Eqn. 24, it will yield:

$$r_{xy} = \frac{A}{\sqrt{B.c}}$$

Where:

$A = 32 * 81.7521 - (36.5451)(38.42)$

$B = [32 * 81.7647 - (36.5451)^2]$

$C = [32 * 86.0653 - (38.42)^2]$

Thus,

$r_{32\,results} = \ 0.947279$

The thirty-two experimental results measured soil loss and predicted soil loss in Table 5 were used for model verification using coefficient of correlation r, the correlation parameters are shown in Table 6 to confirm the goodness of fit of the two results of soil loss value. That is the comparison of the measured and predicted results with coefficient of Correlation, r as0.947279.

The non-highlighted experiments from Table 5 were tabulated for the computation of correlation coefficient in Table 7.

Table 7: Other Experimental Soil loss Measured and Predicted

S/ No.	Exp. No.	Measured values (x)	Predicted values (y)	xy	x^2	y^2
1	2	0.576	0.557	0.321	0.332	0.310
2	3	0.542	0.453	0.245	0.294	0.205
3	4	2.86	1.295	3.703	8.180	1.677
4	5	1.508	1.212	1.827	2.274	1.468
5	8	0.575	1.230	0.707	0.331	1.512
6	9	3.75	3.352	12.570	14.063	11.236
7	10	3.56	4.084	14.539	12.674	16.679
8	11	3.34	3.411	11.394	11.156	11.637
9	12	0.54	0.350	0.189	0.292	0.123
10	14	0.24	0.183	0.044	0.058	0.033
11	15	0.81	0.923	0.748	0.656	0.852
12	17	1.88	1.939	3.645	3.534	3.759
13	18	1.77	1.959	3.468	3.133	3.839
14	19	1.66	1.676	2.782	2.756	2.808
15	20	0.323	0.224	0.072	0.104	0.050
16	21	0.518	0.487	0.252	0.268	0.237
17	24	0.162	0.123	0.020	0.026	0.015
18	26	0.331	0.351	0.116	0.110	0.123
19	27	0.152	0.164	0.025	0.023	0.027
20	28	1.43	0.599	0.857	2.045	0.359
21	29	0.754	0.536	0.404	0.569	0.287
22	31	0.94	0.953	0.896	0.884	0.909
23	32	1.95	2.211	4.311	3.803	4.888
∑	∑	30.171	28.272	63.135	67.565	63.033

Calculating the correlation result by substituting the function of the Eqn. 24 will yield;

$$r_{xy} = \frac{A}{\sqrt{B.c}}$$

Where:

$A = 23 * 63.135 - (28.272)(30.171)$

$B = [23 * 63.033 - (28.272)^2]$

$C = [23 * 67.565 - (30.171)^2]$

Thus,

$r_{23,Otherresult} = \ 0.925881$

The non-highlighted experimental results that were not used for the regression model development for the soil loss prediction were used in the model verification using the coefficient of correlation, r to confirm the goodness of fit of the two results of soil loss values and the predicted ones. The 23 experiments has shows that their coefficient of Correlation, r is0.925881.

However, Table 8 shows the summary of the calibration and verification of the experimental results for soil loss prediction and measured. Express the values for the maximum of soil loss in the process, the correlation coefficient and coefficient of determination, in the selected 9 experiments,23 and the entire experiments.

Table 8: Calibration of study Models to Soil Loss Prediction

Calibration Verification	Area sqkm	Soil Loss Maximum kg or ton	R	R^2
Cal. ESLRM vs Exp.	9, sele. Exp.	4.0kg	0.99	0.99
Veri. of ESLRM vs Exp.	23, other Exp.	4.1kg	0.93	0.86
Veri. of ESLRM vs Exp.	All the 32 Exp.	4.1kg	0.95	0.90

4.4 Formulated Model Validation Test

Correlation (r) of Empirical Soil Loss Regression Model

The validation test of the model was summarized in Table 8. This is of two categories, calibration and verification. And each of the process was carried out using the correlation coefficient test r and coefficient of determination r^2 which is obtained from a scattered graph trend.

For calibration which is a test of 9 selected test results for which the ESLRM was developed. And their coefficient of correlation r is 0.99 perfect correlations and coefficient of determination r^2 is 0.99 which is a good fit and the maximum soil loss recorded is 4.0kg in the experiment.

While for verification of the model is for the non-selected 23 experiments and the entire 32 experiments including the 9 selected ones. However, their correlation coefficient r is 0.93 and 0.95 respectively. While their coefficient of determination r^2 is 0.86 and 0.90 respectively with the maximum soil loss value of 4.1kg all are represented in Table 8.

5.1Conclusions

Soil loss was measured in the Laboratory for the thirty-two experiments conducted with the experimental parameters including rainfall intensity, slope of the catchment, duration of the runoff/rain, density of soil, catchment area, organic content and clay content. Nine experiments selected out off the thirty-two experiments conducted were used in formulating the Empirical Soil loss Regression Model (ESLRM).

For any catchment, if the soil characteristics are known, the Empirical Soil Loss Regression Model (ESLRM) can be confidently adopted in calculating the amount of soil loss, without running into difficulties and time wasting of field measurements.

Results of model calibration for ESLRM which gave r calculated correlation value of 0.9998544 showed very high correlation. Model verification yielded R calculated value of 0.947279. The two correlation coefficients gave rise to standard percentage error of approximant 5%. The standard error of ± 5% is admissible, so the regression models were found to be adequate for Erosion studies.

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